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DISTRIBUTED MOVING LOAD ON NON-UNIFORM BERNOULLI-EULER BEAM RESTING ON BI-PARAMETRIC FOUNDATIONS

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ABSTRACT

This paper concerned with the dynamic analysis of non-uniform Bernoulli-Euler beam resting on bi-parametric foundations and traversed by constant magnitude moving distributed load with simply supported ends conditions. Damping term effect is incorporated into the model. The solution technique employed is based on Galerkin method and integral transformation in conjunction with the convolution theorem. The deflection of the beam under moving loads is calculated for several values of damping coefficient (ε), shear modulus (G), axial force (N) and foundation modulus (K). The results are shown graphically as a function of time.

KEYWORDS: *Non-Uniform Beam, Bi-Parametric Foundations, Axial Force, Shear Modulus, Foundation Modulus, Damping Coefficient, Galerkin Method, and Simply Supported Conditions*

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INTRODUCTION

The dynamic response of elastic structures subjected to one or more travelling load is an interesting problem in several fields of applied mathematics, engineering and applied physics; and the problem has been studied by many authors [1-10]. However, this beam's problem has largely been restricted to the case when the beam's structure is uniform; the more difficult beam's problem in which the mass per unit length of the beam and the moment of inertia varies with certain function of the spatial coordinate x in the model equation received scanty attention. The difficulty in the latter problem is associated with the variable coefficient which appears in the governing equation describing the dynamical problem.

Among the few researchers on the dynamic analysis of non-uniform beam are Oni and Omolofe [11], Hsu [12], Zheng*et al* [13], Oni and Awodola [14], and Omolofe*et al* [15]. All these studies adopted Winkler elastic foundation and it is well known that Winkler foundation predicts discontinuities in the deflections of the surface of the foundation at the end of a finite beam and in reality, the surface displacement continues beyond the load region.

To overcome this problem, Oni and Jimoh [16, 17] considered dynamic response to moving concentrated loads of non-uniform beam resting on bi-parametric subgrades with simply supported and other boundary conditions respectively. Their results revealed that, the deflection profiles of the beam decreases as the values of foundation modulus, shear modulus and axial force increases. The dynamic analysis of Bernoulli-Euler beam resting on bi-parametric subgrades and subjected to concentrated moving loads for all general boundary conditions has been investigated by Jimoh [18]. He used generalized finite Fourier sine transform and generalized Galerkin's methods of solutions to the governing equations describing the dynamical system. The results represented in graphical manner shows that, increase in the structural parameters lead to decrease in the response amplitudes of the beam. His result also revealed that, the effect of shear modulus is more noticeable compare to that of foundation modulus.

In a recent times, Ogunyebi [19] investigated flexural vibrations of non-uniform Rayleigh beam resting on variable bi-parametric elastic foundation and traversed by moving distributed loads. In this study, damping term effect was not taking into consideration.

This paper therefore, is concerned with the problem of the transverse motion of non-uniform Bernoulli-Euler beam resting on bi-parametric foundations under the action of constant magnitude moving distributed load and taking into consideration the damping effect.

Formulation of the Problem

The governing partial differential equation for a non-uniform Bernoulli-Euler beam of length Lresting on bi-parametric foundations and traversed by a constant magnitude distributed load $P(x,t)$ of mass M moving with constant velocity c is given by [20]

$$\frac{\partial^2}{\partial x^2}\left(EJ(x)\frac{\partial^2 W(x,t)}{\partial x^2}\right)+\mu(x)\frac{\partial^2 W(x,t)}{\partial t^2}+\varepsilon\frac{\partial W(x,t)}{\partial t}-N\frac{\partial^2 W(x,t)}{\partial x^2}+F_R(x,t)=P(x,t) \qquad (1)$$

where$x$is the spatial coordinate, t is the time, $W(x,t)$ is the transverse displacement,$EJ(x)$ is the variable flexural rigidity of the structure, $\mu(x)$ is the variable mass per unit length of the beam, N is the axial force,, ε is the damping coefficient, $F_R(x,t)$ is the foundation reaction and $P(x,t)$ is the transverse distributed load.

The foundation reaction$F_R(x,t)$is given by Omer and Aitung [21]

$$F_R(x,t)=KW(x,t)+G\frac{\partial^2 W(x,t)}{\partial x^2} \qquad (2)$$

Where $K$and G are the foundation stiffness and shear modulus respectively.

If the distributed moving load $P(x,t)$ in equation (1) is assumed to be of constant magnitude we thus have

$$P(x,t)=PH(x-ct) \qquad (3)$$

The Heaviside function$H(x-ct)$ is defined as

$$H(x-ct)=\begin{cases}0, & for\ \ x<0\\ 1, & for\ \ x>0\end{cases} \qquad (4)$$

with the properties

$$\frac{d}{dx}\{H(x-ct)\}=\delta(x-ct) \qquad (5)$$

$$f(x)H(x-ct)=\begin{cases}0, & for\ x<ct\\ f(x), & for\ x\geq ct\end{cases} \tag{6}$$

where $\delta(x-ct)$ represents the Dirac-delta function and $H(x-ct)$ is a typical engineering

function made to measure engineering applications.

Adopting the example in [22], $\mu(x)$ and $J(x)$ are taken to be of the form

$$\mu(x)=\mu_0\left(1+Sin\frac{\pi x}{L}\right) \tag{7}$$

$$J(x)=I_0\left(1+Sin\frac{\pi x}{L}\right)^3 \tag{8}$$

In this study, a simply supported beam is considered, thus, the boundary conditions are

$$W(0,t)=0=W(L,t),\frac{\partial W(0,t)}{\partial x}=0=\frac{\partial W(L,t)}{\partial x} \tag{9}$$

and the initial conditions

$$W(x,0)=0=\frac{\partial W(x,0)}{\partial t} \tag{10}$$

Using (2), (3), (7) and (8) in equation (1) we have

$$\frac{\partial^2}{\partial x^2}\left(E\,I_0\left(1+Sin\frac{\pi x}{L}\right)^3\frac{\partial^2 W(x,t)}{\partial x^2}\right)+\mu_0\left(1+Sin\frac{\pi x}{L}\right)\frac{\partial^2 W(x,t)}{\partial t^2}+\varepsilon\frac{\partial W(x,t)}{\partial t}-N\frac{\partial^2 W(x,t)}{\partial x^2}$$

$$-G\frac{\partial^2 W(x,t)}{\partial x^2}+KW(x,t)=PH(x-ct) \tag{11}$$

which after simplifying and rearrangement yields

$$\frac{\partial^2}{\partial x^2}\left(E\,I_0\left(10+15Sin\frac{\pi x}{L}-6Cos\frac{2\pi x}{L}-Sin\frac{3\pi x}{L}\right)\frac{\partial^2 W(x,t)}{\partial x^2}\right)+\mu_0\left(1+Sin\frac{\pi x}{L}\right)\frac{\partial^2 W(x,t)}{\partial t^2}+\varepsilon\frac{\partial W(x,t)}{\partial t}-(N+G)\frac{\partial^2 W(x,t)}{\partial x^2}$$

$$+KW(x,t)=PH(x-ct) \tag{12}$$

Equation (12) is the fourth order partial differential equation governing the motion of a non-uniform Bernoulli-Euler beam resting on bi-parametric foundations and traversed by constant magnitude moving distributed load with damping effect.

Method of Solution

The best method suited for solving diverse problems involving mechanical vibration [23,24] is referred to as Galerkin's method. This method is used to simplify and reduce the fourth order partial differential equation (12) with variable coefficient describing the motion of the vibrating non-uniform to second order ordinary differential equations called Galerkin's equation.

$$W_j(x,t)=\sum_{j=1}^{n}Y_j(t)q_j(x) \tag{13}$$

where the function $q_j(x)$ is chosen to satisfy the pertinent boundary conditions.

Thus, substituting equation (13) into equation (12), we obtain

$$\sum_{j=1}^{n}\left[\frac{EI_0}{4}\frac{\partial^2}{\partial x^2}\left(\left(10+15Sin\frac{\pi x}{L}-6Cos\frac{2\pi x}{L}-Sin\frac{3\pi x}{L}\right)q_j^{11}(x)Y_j(t)\right)\right.$$

$$+\mu_0\left(1+Sin\frac{\pi x}{L}\right)q_j(x)\ddot{Y}_j(t)+\varepsilon\, q_j(x)\dot{Y}_j(t)-(N+G)q_j^{11}(x)\,Y_j(t)$$

$$\left.+K\,q_j(t)Y_j(t)\right]-PH(x-ct)=0 \quad (14)$$

To determine $Y_j(t)$, the expressions on the left hand sides of equation (14) is required to be orthogonal to the function $q_j(t)$. Thus,

$$\int_0^L\sum_{j=1}^{n}\left[\frac{EI_0}{4}\frac{\partial^2}{\partial x^2}\left(\left(10+15Sin\frac{\pi x}{L}-6Cos\frac{2\pi x}{L}-Sin\frac{3\pi x}{L}\right)q_j^{11}(x)Y_j(t)\right)\right.$$

$$+\mu_0\left(1+Sin\frac{\pi x}{L}\right)q_j(x)\ddot{Y}_j(t)+\varepsilon\, q_j(x)\dot{Y}_j(t)-(N+G)q_j^{11}(x)\,Y_j(t)$$

$$\left.+K\,q_j(t)Y_j(t)\;-\;PH(x-ct)\right]q_k(x)dx=0 \quad (15)$$

Equation (15) after some rearrangement and simplifying yield.

$$Q_1\ddot{Y}_j(t)+Q_2\dot{Y}_j(t)+Q_3Y_j(t)=Q_4 \quad (16)$$

Where

$$Q_1=\mu_0\int_0^L\left(1+Sin\frac{\pi x}{L}\right)q_j(x)q_k(x)\,dx \quad (17a)$$

$$Q_2=\varepsilon\int_0^L q_j(x)q_k(x)\,dx \quad (17b)$$

$$Q_3=\int_0^L\frac{EI_0}{4}\frac{\partial^2}{\partial x^2}\left(\left(10+15Sin\frac{\pi x}{L}-6Cos\frac{2\pi x}{L}-Sin\frac{3\pi x}{L}\right)q_j^{11}(x)q_k(x)\right)dx \quad (17c)$$

$$Q_4=\int_0^L PH(x-ct)q_k(x)\,dx \quad (17d)$$

Since our beam has simple supports at both ends $x=0$and $x=L$, we therefore choose the functions $q_j(x)$and $q_k(x)$ to be

$$q_j(x)=Sin\frac{j\pi x}{L} \quad (18)$$

$$q_k(x)=Sin\frac{k\pi x}{L} \quad (19)$$

In view of (18) and (19) integrals (17a-17d) for $j = K$ can be evaluated to yields

$$Q_1 = \mu_0\left(\frac{L}{2} - \frac{L}{2\pi}\left(2(Cos\pi - 1) - \frac{Cos(1+2j)\pi - 1}{1+2j} - \frac{Cos(1-2j)\pi - 1}{1-2j}\right)\right) \tag{20a}$$

$$Q_2 = \frac{\varepsilon L}{2} \tag{20b}$$

$$Q_3 = \frac{EI_0}{4}\left(\frac{j\pi}{L}\right)^4\left[-\frac{15L}{4\pi}\left(2(Cos\pi - 1) - \frac{Cos(1+2j)\pi - 1}{1+2j} - \frac{Cos(1-2j)\pi - 1}{1-2j}\right)\right.$$
$$-\left(\frac{L}{3\pi} + \frac{L}{3\pi}\left(\frac{Cos(3+2j)\pi - 1}{3+2j} - \frac{Cos(3-2j)\pi - 1}{3-2j}\right)\right) - \frac{3L}{2\pi}\left(Sin2\pi + \frac{Sin2(1+j)\pi - 1}{2(1+j)}\right.$$
$$\left.\left. - \frac{Sin2(1-j)\pi - 1}{2(1-j)}\right)\right]$$
$$-\frac{EI_0}{4}\left(\frac{j\pi}{L}\right)^2\left[\frac{9\pi L}{2}\left(\frac{2}{3} - \frac{Cos(3+2j)\pi - 1}{6+4j} - \frac{Cos(3-2j)\pi - 1}{6-4j}\right)\right.$$
$$-\frac{15\pi}{4}\left(2Cos\pi - 2 + \frac{Cos(1+2j)\pi - 1}{(1+2j)} - \frac{Cos(1-2j)\pi - 1}{(1-2j)}\right)$$
$$\left.+\frac{6\pi}{L}\left(Sin2\pi - \frac{Sin2(1+j)\pi - 1}{2(1+j)} - \frac{Sin2(1-j)\pi - 1}{2(1-j)}\right)\right] + \frac{L(G+N)}{2L}\left(\frac{j\pi}{L}\right)^2 + \frac{KL}{2} \tag{20c}$$

$$Q_4 = \frac{PL}{K\pi}\left(Cos\frac{j\pi ct}{L} - Cosj\pi\right) \tag{20d}$$

Putting (20d) in equation (14) we obtain

$$Q_1\ddot{Y}_j(t) + Q_2\dot{Y}_j(t) + Q_3Y_j(t) = \frac{PL}{K\pi}\left(Cos\frac{j\pi ct}{L} - Cosj\pi\right) \tag{21}$$

Equation (21) can be re-written as

$$\ddot{Y}_j(t) + C_{11}\dot{Y}_j(t) + C_{12}Y_j(t) = C_{13}(Cos\alpha t - \beta) \tag{22}$$

where $C_{11} = \frac{Q_2}{Q_1}, C_{12} = \frac{Q_3}{Q_1}, C_{13} = \frac{PL}{Q_1K\pi}, \quad \alpha = \frac{j\pi c}{L}, \beta = Cosj\pi$ (23)

In what follows we subject the system of ordinary differential equation (22) to a Laplace transform defined as

$$\mathcal{L}\left(Y(t)\right) = \int_0^L Y(t)e^{-st}\,dt = Y(s) \tag{24}$$

where S is the Laplace parameter. Applying the initial conditions (10), we obtain

$$S^2Y_j(t) + C_{11}SY_j(t) + C_{12}Y_j(t) = C_{13}\left(\frac{S}{S^2+\alpha^2} - \beta\left(\frac{1}{S}\right)\right) \tag{25}$$

After simplifying and rearrangement equation (25) take the form

$$Y_j(S) = \frac{C_{13}}{n_1+n_2}\left[\left(\frac{1}{S-n_1}\right)\left(\frac{S}{S^2+\alpha^2}\right) - \left(\frac{1}{S-n_2}\right)\left(\frac{S}{S^2+\alpha^2}\right) - \left(\frac{1}{S-n_1}\right)\beta\left(\frac{1}{S}\right) + \left(\frac{1}{S-n_2}\right)\beta\left(\frac{1}{S}\right)\right] \tag{26}$$

where

$$n_1 = -\frac{C_{11}}{2} + \frac{\sqrt{C_{11}^2 - 4C_{12}}}{2} \tag{27}$$

$$n_2 = -\frac{C_{11}}{2} - \frac{\sqrt{C_{11}^2 - 4C_{12}}}{2} \tag{28}$$

To obtain the Laplace inversion of equation (26), use is made of the following representations

$$f_1(S) = \frac{S}{S^2+\alpha^2}, \quad f_2(S) = \frac{1}{S} \tag{29}$$

$$g_1(S) = \left(\frac{1}{S-n_1}\right), \quad g_2(S) = \left(\frac{1}{S-n_2}\right) \tag{30}$$

So that the Laplace inversion of equation (26) is the convolution of f_i's and g_i's defined as

$$f_i * g_j = \int_0^{\omega} f_i(t-u) g_i(u) du, \quad i = 1,2$$

$$j = 1,2 \tag{31}$$

Thus, the Laplace inversion of (26) is given by

$$Y_j(t) = \frac{C_{13}}{(n_1+n_2)(n_1^2+\alpha^2)} (n_1 e^{n_1 t} + \alpha\, Sin\alpha t - n_1\, Cos\alpha t +)$$

$$- \frac{C_{13}}{(n_1+n_2)(n_2^2+\alpha^2)} (n_2 e^{n_2 t} + \alpha\, Sin\alpha t - n_2\, Cos\alpha t +)$$

$$+ \frac{\beta C_{13}}{n_1(n_1-n_2)} (1 - e^{n_1 t}) - \frac{\beta C_{13}}{n_1(n_1-n_2)} (1 - e^{n_2 t}) \tag{32}$$

Thus, in view of equation (13) taking into account equation (32) we obtain

$$W_j(x,t) = \sum_{j=1}^{\infty} \left[\frac{C_{13}}{(n_1+n_2)(n_1^2+\alpha^2)} (n_1 e^{n_1 t} + \alpha\, Sin\alpha t - n_1\, Cos\alpha t +) \right.$$

$$- \frac{C_{13}}{(n_1+n_2)(n_2^2+\alpha^2)} (n_2 e^{n_2 t} + \alpha\, Sin\alpha t - n_2\, Cos\alpha t +)$$

$$\left. + \frac{\beta C_{13}}{n_1(n_1-n_2)} (1 - e^{n_1 t}) - \frac{\beta C_{13}}{n_1(n_1-n_2)} (1 - e^{n_2 t}) \right] Sin \frac{j\pi x}{L} \tag{33}$$

which represent the transverse displacement response of the non-uniform Bernoulli-Euler beam resting on bi-parametric foundations under the action of constant magnitude moving distributed load.

Discussion on the Close Form Solution

In this section, we investigated the resonance phenomenon of our vibrating system, because the transverse displacement of the beam may grow without bound. It is clearly shown from equation (33) that the non-uniform Bernoulli-Euler beam resting on bi-parametric foundation will experience resonance effects whenever

$$n_1^2 = \alpha^2,\ n_2^2 = \alpha^2, \text{ and } n_1 = n_2 \tag{34}$$

It is also observed that as the foundation modulli and shear modulus increases the critical speed of the dynamical system increases thereby reducing the risk of resonance effects.

Comments on the Numerical Results

In this paper for the purpose of analysis, we illustrated the theory numerically. The velocity of the distributed moving load and the length of the beam are c = 8.128 m/s and 12.192m respectively. The values of the damping coefficient (ε) are varied between 0 and 2.5×10^4 while that of the shear modulus (G) varied between 0 and $2 \times 10^6 N/m^3$. The axial force (N) varied between 0 and $2 \times 10^6 N/m^3$ while that of foundation modulus (K) varied between 0 and $4 \times 10^5 N/m^3$. The results are presented in the graphs below.

Figure 1 displays the deflection profile of non-uniform Bernoulli-Euler beam resting on bi-parametric foundation and subjected to constant magnitude moving distributed load. The figure shows that as the value of the damping coefficient (ε) increase the deflection profile of the beam at various time t decreases.

Figure 2 depicts the deflection profile of the beam under the action of moving distributed load. It is seen from the figure that the response amplitude of the beam decreases with an increase in the values of the shear modulus (G).

Figure 3 and 4 shows respectively that as we increase the axial force (N) and foundation modulus (K), the transverse displacement response of the non-uniform Bernoulli-Euler beam under the action of moving load reduces.

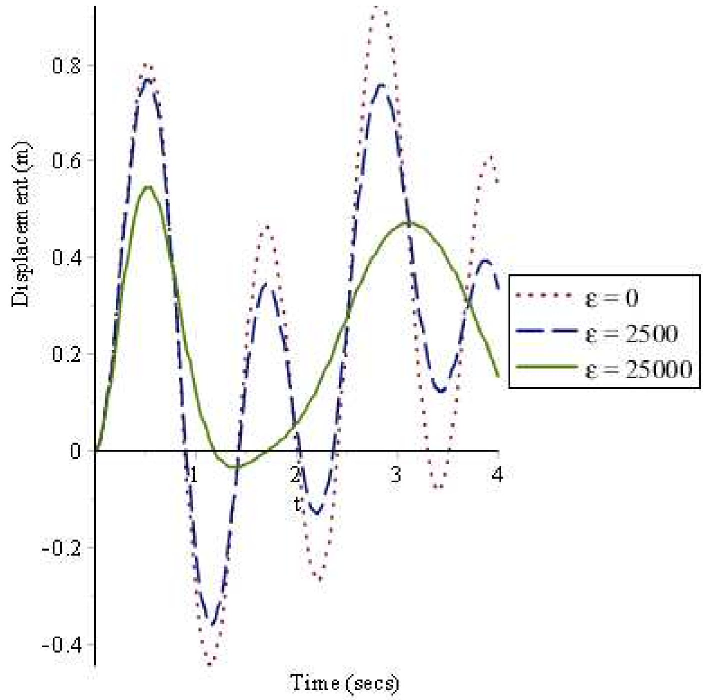

Figure 1: Deflection Profile of Non-Uniform Bernoulli-Euler Beam Subjected to Constant Distributed Moving Load for Fixed Value of Axial Force (N), Shear Modulus (G) and Foundation Modulus (K) with Various Values of Damping Coefficient (ε).

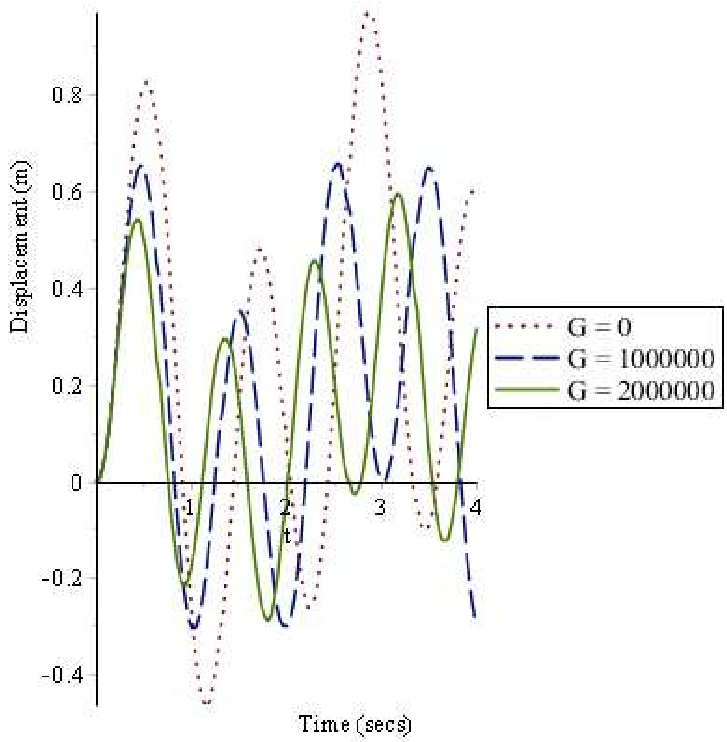

Figure 2: Deflection Profile of Non-Uniform Bernoulli-Euler Beam Subjected to Constant Distributed Moving load for Fixed Value of Axial Force (N), Damping Coefficient (ε) and Foundation Modulus (K) with Various Values of Shear Modulus (G).

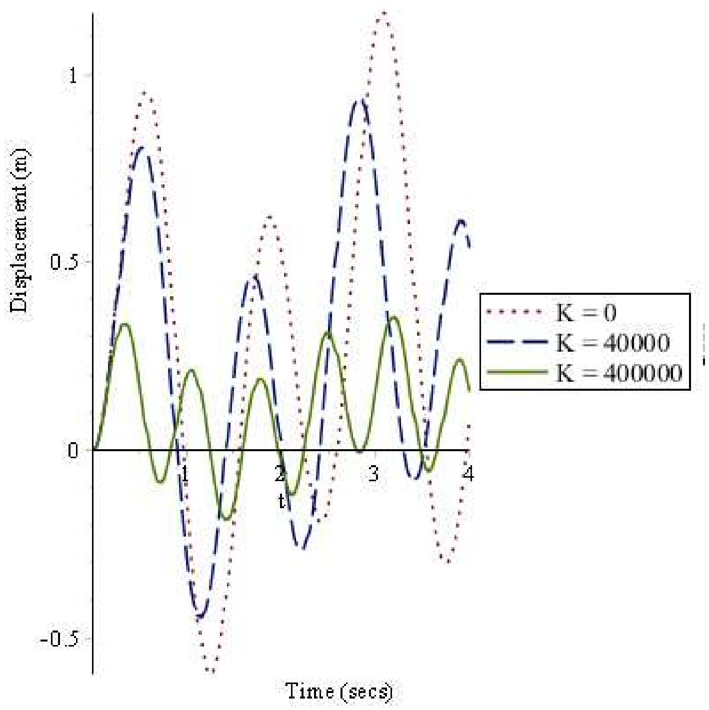

Figure 3: Deflection Profile of Non-Uniform Bernoulli-Euler Beam Subjected to Constant Distributed Moving Load for Fixed Value Of Damping Coefficient (ε), Shear Modulus (G) and Axial Force (N) with Various Values of Foundation Modulus (K).

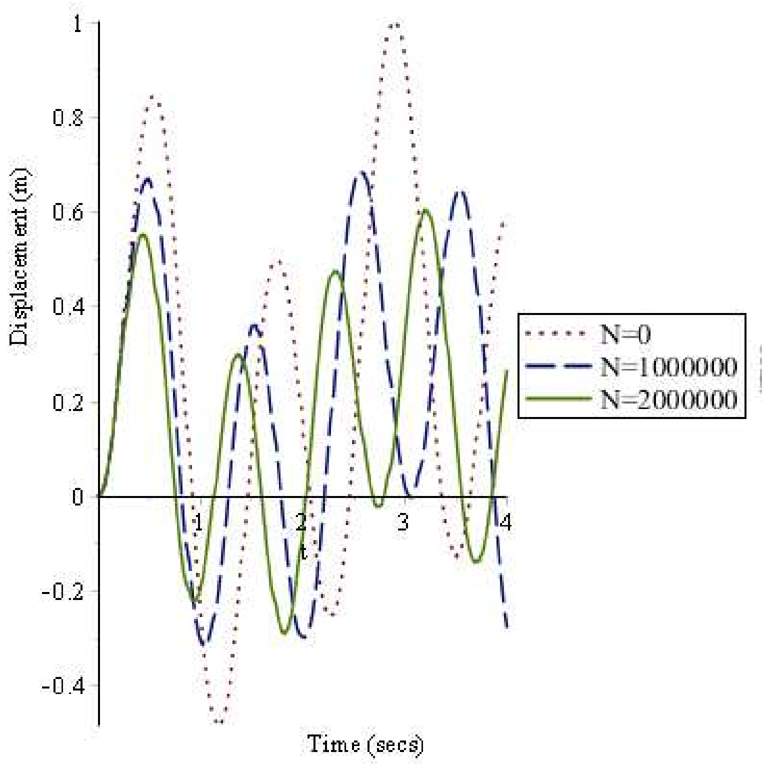

Figure 4: Deflection Profile of Non-Uniform Bernoulli-Euler Beam Subjected to Constant Distributed Moving Load for Fixed Value of Damping Coefficient (ε), Shear Modulus (G) and Foundation Modulus (K) with Various Values of Axial Force (N).

CONCLUSIONS

The objective of this work is to study the behavior of the dynamical system. A procedure involving the Galerkins method and integral transform techniques in conjunction with the convolution theory has been used to obtain an analytical solution in series form. The effects of damping coefficient (ε), shear modulus (G), foundation modulus (K) and the axial force (N) on the vibrating system are investigated. Analytical solution and Numerical results in plotted curves show that, as the values of those structural parameters increases, the deflection profile of the non-uniform Bernoulli-Euler beam decreases. Thus, the risk of resonance in the dynamical system under consideration reduces for higher values of damping coefficient in particular and other structural parameters.

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